Measuring the Solar Constant and the Sun's Temperature

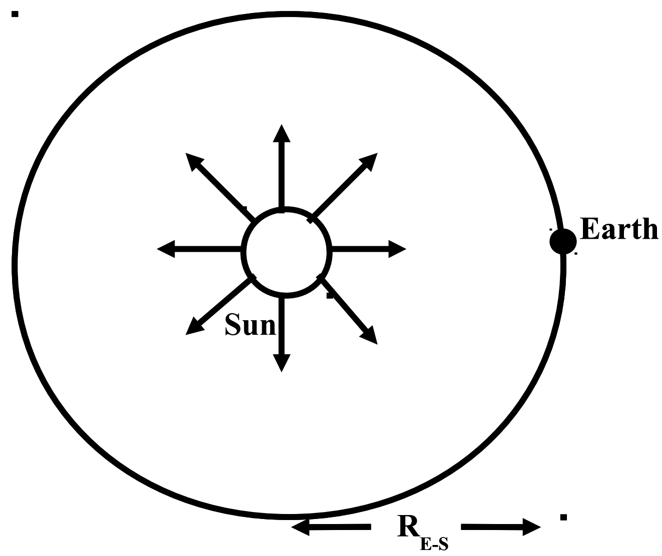
The principles involved

1. The Stefan-Boltzmann Law

According to this law, each square meter of the Sun's surface radiates σT_S^4 Joules of energy per second, where T_S is the surface temperature of the Sun and $\sigma = 5.67 \times 10^{-8} \frac{J}{m^2 \cdot s \cdot K^4}$. The temperature is measured in Kelvins (K) which is related to the more familiar Celsius temperature by: K = C + 273. The Sun's surface temperature is about $T_S = 5800$ K.

The number of square meters on the Sun is what we mean by its "surface area," namely $4\pi R_S^2$.

Because this energy is radiated outward from the Sun’s surface, the total amount of energy emitted by the Sun each second equals the energy emitted by each square meter times the number of square meters of the Sun, i.e., The total energy leaving the Sun’s surface per second = $\sigma T_S^4 \times 4\pi R_S^2$ Joules per second.



2. *Conservation of energy*

A small part of this light-energy, called "solar energy", from the Sun strikes the top of the Earth's atmosphere. The law of conservation of energy says that the same total energy radiated by the Sun per second must equal the total energy passing through the sphere centered at the sun and containing the Earth's orbit (See the figure above.). The radiated energy spreads out or "diverges." Thus, there will be much less energy passing through a square meter per second at the Earth's orbit than the value σT_S^4 at the Sun's surface.

The solar <u>cons</u>tant

The amount of solar energy falling on the earth's outer atmosphere per second per square meter is called the "solar constant" and is represented by the letter, S. In other words, the solar constant is the intensity of the sunlight falling on the earth's outer atmosphere. By conservation of energy, energy per second through the Earth-orbit sphere = Energy per second radiated by the Sun or

$$S \times 4\pi R_{E-S}^2 = \sigma T_S^4 \times 4\pi R_S^2$$

Therefore, the solar constant, S, and the Sun's surface temperature, T_S , are related by

$$S = \sigma (R_S/R_{E-S})^2 T_S^4$$

This is even simpler than it appears because the ratio, R_S/R_{E-S} , is just half of the angular diameter of the Sun as seen from the Earth. Let θ denote this angular diameter, measured in radians (1 radian = 57.3 degrees of angle). Thus,

$$S = \sigma(\theta/2)^2 T_S^4$$

or

$$T_S = \left(\frac{4S}{\sigma\theta^2}\right)^{1/4} \qquad (1)$$

Summary of the ideas above

According to Eq. 1, the surface temperature of the Sun can be determined by measuring only two things: (1) the solar constant, and (2) angular diameter of the Sun.

In this project, you will measure these two quantities and thereby determine the Sun's surface temperature! Who would have thought it could be done so easily?!

Determining the solar constant

The solar constant is determined by measuring the temperature rise of a known amount of water in a black pan in a given length of time when the pan of water is placed in full sunlight on a clear day. This experiment does not include several corrections that, when taken into consideration, yield better results, complicate the calculation, and do not contribute significantly to understanding the physics of this exercise. We will therefore, not explicitly take them into account.

The solar constant is given by

$$S = \frac{mC_p}{A}\frac{d}{h}\frac{\Delta T}{\Delta t}. \qquad (2)$$

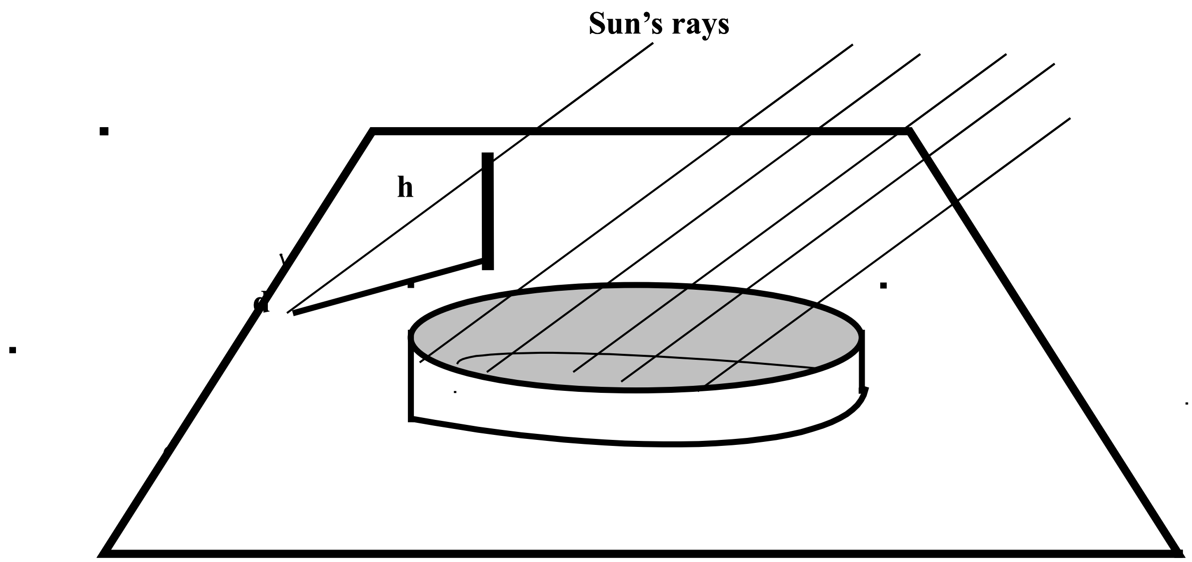
For Eq. 2, m is the mass of the water in the pan, C_p is the specific heat of water, A is the cross sectional area of the pan, h is the height of a stick and d is the distance from the top of the stick to the end of the shadow cast by the stick, and $\Delta T/\Delta t$ is the rate of temperature rise during the experiment, which is ultimately the slope of a line that you will plot. This ignores the heat capacity of the pan, but it is quite negligible for disposable pans.

The specific heat of water is: C_p = 4184 J/kg °C

<u>Equipment needed</u>:

A sheet of plywood with a pencil-sized hole near one edge
A stick (to fit into the hole)
A cake pan, painted black on the inside
A thermometer (use the one provided with a sensor probe)
A watch
A small mirror
A sheet of paper with a 1/4" hole in the center (this is the "pinhole")

A ruler (various measurements)
A tape measure (to measure the distance from pinhole to the Sun's image)



Preparing the equipment
(See the figure above.)

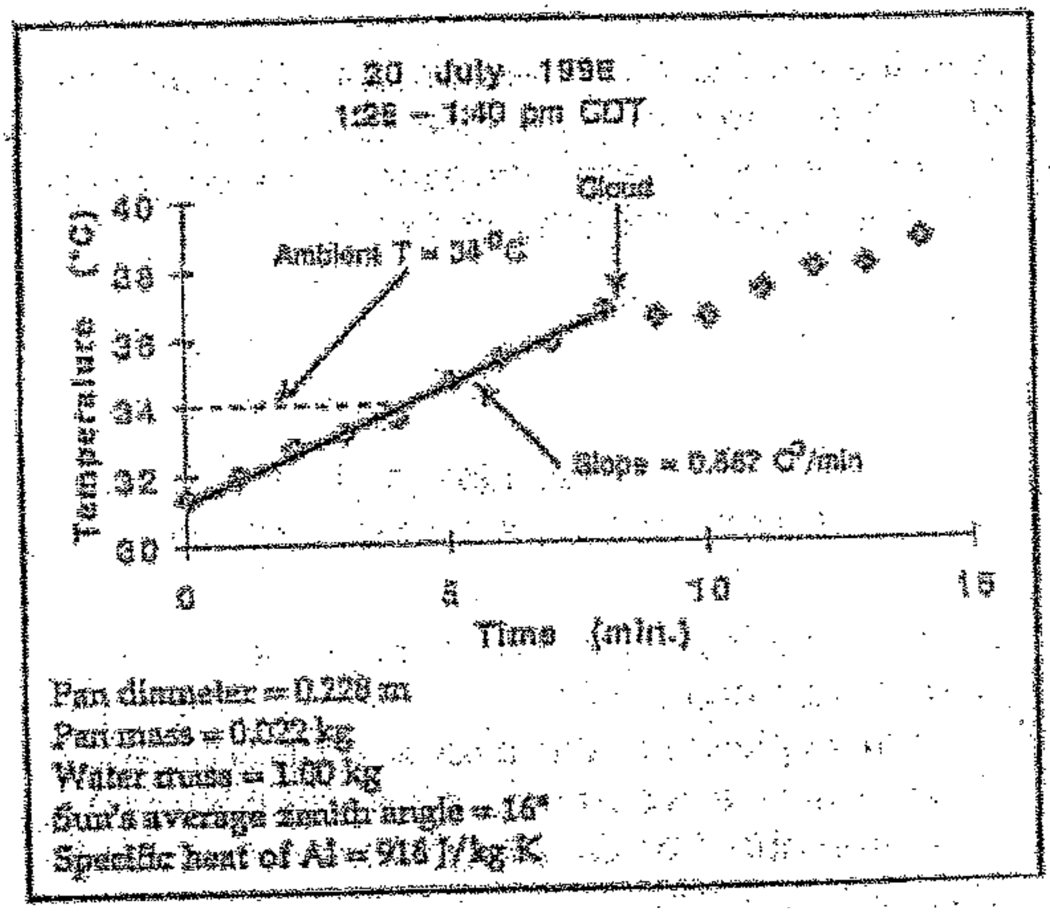
- Prime, then paint the inside of a disposable aluminum cake pan with flat black paint. Keep the outside of the pan as shiny as possible.
- Drill a hole accurately perpendicular to and near the edge of a piece of plywood about one foot square. Insert a pencil, pointed at the top, into the hole. This will be used to measure the zenith angle of the Sun at the beginning and end of the measurement period.
- Cut a small hole in a piece of paper and tape to the surface of a mirror. This will be used to measure the angular diameter of the Sun.

Taking the data

- Tape a sheet of paper to the plywood and mark the tip of the pencil's shadow. Measure the diameter of the cake pan and calculate the area. $A = \pi(D/2)^2$
- Measure d, the distance from the top of the stick to the end of the stick's shadow, and h the height of the stick.
- Place the cake pan on plywood and pour a measured amount of water into the pan. One liter makes a nice amount for a 9-inch cake pan. The water temperature should be about 5° C below ambient air temperature. Tap water from a cold faucet should suffice.
- Place the thermometer probe in the water: set the switch on the back to "°C" *Note:* If you place the thermometer readout unit in the shade, you can use the switch on the front to read off either the probe temperature in the water ("outside") or the ambient air temperature ("inside").
- Lay a sheet of Saran wrap over the pan/water. Let the Saran wrap rest directly on the water surface (fewer surfaces to reflect light).
- Record the temperature every minute over the temperature range.
- Take the data until the water temperature has risen above the air temperature an amount equal the difference between the water and the air at the start. For example, if your water was 5˚C below the air temperature, you should end your experiment when the water is 5˚C above the air temperature. You should have the same difference below as you do above. (This reduces the effect of heat conduction through the pan walls.)
- Remove the pan of water and measure d again. Determine an average value for d by using the before and after measurements.
- To obtain the Sun's angular diameter, reflect sunlight off of the prepared mirror and measure the Sun's image at a known distance from the mirror: 10 feet =120 inches is a good distance. The angular diameter is θ = diameter/distance (in radians).

Plot your data as in the figure below, which shows data taken at Little Rock, AR. The data segment shown, centered on the air temperature, was used to determine $\Delta T / \Delta t$. A small cloud caused the dip just afterwards. These data give S =1196 J/s m2 compared to the accepted value, 1370 J/s m2

The pinhole measurement gave the Sun's angular diameter = 0.00845 radian (0.484°). Combined with the measurement of S, this gives T_S = 5840 K. The book value is 5780 **K.**



Calculating your results

- Use Eq. 2 to calculate S from your data. Note that $\Delta T/\Delta t$ is just the slope of your graph, using the part centered on ambient temperature as shown in the figure above. To measure this slope, draw the best straight line through your data measure its slope.
- The angular diameter of the Sun is $\theta = \dfrac{\text{image diameter}}{\text{mirror to image distance}}$. *Note:* Use the same units for both measurements. Do *not,* for example, use inches for the diameter and feet for the distance.
- Use the *average* length of the pencil's shadow to determine the average zenith angle, φ, as described above.
- Use Eq. 2 to Calculate S.
- Use Eq. 1 to calculate the surface temperature of the Sun.